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Matter asymmetry sourced dark matter

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We investigate the possibility that the dark matter abundance is sourced by the baryon/lepton asymmetry of the early Universe. It turns out that a Goldstone field of a global classically preserved symmetry in the Standard Model experiences a kick during a period of baryon/lepton number generation. This mechanism can be regarded as dynamical generation of initial conditions for misalignment yet the prediction for relic abundance presents an inverse dependence on the coupling in parallel with freeze-in vs freeze-out. We explore two realizations of this mechanism and show that in conjunction with leptogenesis, it is possible to identify a viable promising region of parameter space for dark matter production with mass 10 MeV–1 GeV and decay constant f in the range of 10^{10} – 10^{12} GeV.

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I. INTRODUCTION

Our Universe shows a surprising coincidence: the mass densities of its matter components, baryonic matter and dark matter (DM), are eerily similar down to a factor of four or so. This is odd because their abundance seems to be set by different, often unrelated, mechanisms. The baryon abundance is set by a baryon number asymmetry, whose origin is unknown to us. On the other hand, the dark matter abundance can be set by any number of mechanisms—freeze-out, freeze-in, misalignment, number asymmetry (shared or not shared with the Standard Model). At the end, all of the above mechanisms have enough free parameters that for a range of couplings and DM masses, they can generate just the right amount of DM.

This work will study a possible connection between asymmetry generation and dark matter production. This interplay has already been explored in the literature: the initial ideas were discussed in the 90s [1–3] and many more were published during a resurgence of this topic about a decade or two later, amongst those were [4–9]. Of particular interest to us are models in which a condensate triggers baryogenesis [10,11] while being itself the dark matter [12].

In this work we will present a different explicit realization of this connection.

The one mainstay of DM models is that they should explain why we have not seen DM in any nongravitational experimental apparatus so far. We are going to circumvent this (somewhat serious) constraint by coupling the DM to a divergence of a (mostly) conserved current of the Standard Model (SM) such as baryon number through the operator:

$$\mathcal{L}_{\text{int}} = \frac{\phi}{f} \nabla_\mu J_Q^\mu, \quad (1)$$

where the symmetry and the current, to be explicit, are defined by

$$U(1)_Q: Q = \cos(\alpha)B - \sin(\alpha)L, \quad (2)$$

$$J_Q^\mu = -\frac{\partial \mathcal{L}(G\psi)}{\partial_\mu \theta} \quad G = e^{iQ\theta(x)}. \quad (3)$$

As a result, the DM and the baryons (or leptons) only interact when Q -number is being violated, such as during baryogenesis and in extremely rare events today (which we haven't observed yet).

There are two ways one can generate DM density this way. A quick change of basis, as we will see, turns the classic low dimension baryon/lepton number violating operators into interactions terms between SM and the DM:

$$-\frac{i\phi}{f} \left(-2s_\alpha \frac{y_N^2}{2M_N} (\ell H)^2 + (c_\alpha - s_\alpha) \frac{y_X^2}{M_X} qq\ell \right), \quad (4)$$

and thanks to these the DM production proceeds through a UV-sensitive freeze-in.

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However, there is another curious way in which the SM plasma can transfer its energy into the dark sector. During lepto/baryogenesis $\nabla_\mu J_Q^\mu \neq 0$ so the ϕ field feels a source that, under favorable conditions does work on the ϕ field and deposits energy into its oscillations, hence generating dark matter density [13]. For reasons that will become apparent we will call this mechanism the “kick-alignment”—the main focus of this work.

First, the dynamics of our candidate are discussed in Sec. II, which contain the description of the highlighted production mechanism in II A and comparison with thermal production in II E. Additional computational details can be found on the Appendixes A and B. Two explicit realizations for this mechanism can be found in Sec. III, and we finish with discussion and conclusions in Sec. IV.

II. THE MECHANISM

The coupling of Eq. (1) is indeed most naturally and familiarly realized if dark matter is the (pseudo) Goldstone boson of the Q -symmetry. Let us write the action as

$$S_{\text{eff}} = \int \sqrt{|g|} d^4x \left(\frac{1}{2} (\partial\phi)^2 + m^2 f^2 \cos\left(\frac{\phi}{f}\right) - \frac{\partial_\mu \phi}{f} J_Q^\mu + \mathcal{L}_Q + \mathcal{L}_{\mathcal{Q}} \right), \quad (5)$$

where m is the mass and f the decay constant that gives the periodicity of the field $\phi + 2\pi f$ and the last piece is the effective action that violates this symmetry. In its absence the coupling of ordinary matter to ϕ vanishes since so does $\partial_\mu J_Q^\mu$. This can be seen most explicitly by the field dependent transformation:

$$G(\phi) = e^{-iQ\phi/f} \\ \delta\mathcal{L}_Q(G\psi) = \frac{\partial_\mu \phi}{f} J_Q^\mu, \quad (6)$$

which cancels out the derivative coupling in the first line of Eq. (5). To leading order in $1/f$, the nonderivative fermion operators turn into:

$$\psi_1 \dots \psi_n \rightarrow \psi_1 \dots \psi_n - \frac{i\phi}{f} \left(\sum_i Q_i \right) \psi_1 \dots \psi_n, \quad (7)$$

which makes it obvious only explicitly Q -violating operators lead to nonzero couplings with ϕ . It is important to remark that, unless we consider an exact symmetry in the SM (e.g., $Q = B - L$), there is a Q contribution in the effective action from SM physics.

The case in which this violating term is given by the SM sphaleron processes is an illustrative example. The zero temperature contribution is very much suppressed by the nonperturbative nature of the effect, whereas at high

temperature, the violation is thermal and unpressed. This allows for its production at high energy while simultaneously it leads to a very weak coupling today and hence cosmic scale stability of the DM.

Similar consequences follow if one considers $Q = L$ and the Seesaw model, that is, the Majoron: at high energy the mass term of heavy right handed neutrinos breaks L whereas at low energies the breaking and coupling comes instead from the tiny left handed ν masses [14,15]. Our scenario will indeed present parallels with the Majoron as will be made explicit.

A. Kick-alignment

In this section we show how to calculate the ρ_ϕ due to kick-alignment. Consider the cosmological evolution of the field ϕ ; in an isotropic, homogeneous primordial universe. The dynamics of the field ϕ are, expanding the potential and retaining the mass term only,

$$\frac{d^2\phi}{dt^2} + 3H(t) \frac{d\phi}{dt} + (k^2 + m^2)\phi = \frac{1}{fa^3} \frac{d}{dt}(n_Q a^3), \quad (8)$$

where the rhs shows quantitatively how the breaking of Q -symmetry acts as a source for ϕ , with

$$n_Q \equiv J_Q^0 = \sum_i Q_i (n_{\psi_i} - n_{\bar{\psi}_i}), \quad (9)$$

so for baryon number one has $n_B = n_b - n_{\bar{b}}$. In the following we focus on the zero mode ($k = 0$) which is the last to cross the horizon and will dominate the density; see Fig. 1. Given the solution to the homogeneous equation \hat{J} , the solution to the equations of motion (EoM) with a source is

$$\phi(t) = \hat{J}(t) \left(1 + \int_{t_i}^t \frac{dt'}{a^3 \hat{J}^2} \int_{t_i}^{t'} dt'' \frac{\hat{J}}{f} \frac{d(a^3 n_Q)}{dt''} \right), \quad (10)$$

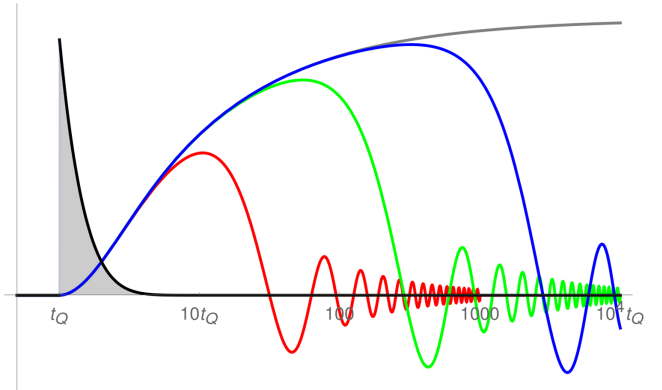


FIG. 1. Black is the source term $d(a^3 n_{\Delta Q})/dt/a^3$, whereas blue, green, and red are $\phi(t)$ solutions for $2r_m^2 = t_Q m = (0.001, 0.01, 0.1)$ and $r_\tau = 1$. Note that for nonzero modes the evolution is obtained from the above by $m^2 \rightarrow m^2 + k^2$.

where the initial conditions $\phi(t_i)$, $\dot{\phi}(t_i)$ are encoded in \hat{J} . During the radiation domination era (RD) this function is

$$\hat{J}(t) = C_1 \frac{J_{1/4}(mt)}{(mt)^{1/4}} + C_2 \frac{Y_{1/4}(mt)}{(mt)^{1/4}} \quad (\text{RD}), \quad (11)$$

where J and Y are the Bessel functions of the first and second kind. We note and assume several things:

- (1) Note that the second term of Eq. (10) is invariant under rescaling $\hat{J} \rightarrow \lambda \hat{J}$. This means that the kick acts independently of the initial amount of dark matter already present.
- (2) The initial condition for ϕ and $\dot{\phi}$ is set by inflation/initial misalignment. We will characterize this amount as $\phi(t_i) = \theta_0 f$. Many misalignment scenarios assume $\theta_0 = \mathcal{O}(1)$, but we will allow smaller, fine tuned values of $\theta_0 \ll 1$. This choice determines the value of C_1 .
- (3) Similarly to the misalignment scenarios the inflation tends to set $\dot{\phi}(t_i) \simeq 0$, which in turn implies smallness of C_2 . We will set $C_2 = 0$.
- (4) Baryogenesis happens after inflation (a fairly safe bet).

The solution in Eq. (10) is that of a driven damped oscillator. The presence of an external force on the oscillator is transient and lasts for the period in which the asymmetry is changing. As a result, there are several timescales in play: The Q -number generation starts at t_Q and lasts until t_F (forcing term time). On the other hand, the field starts its oscillations by the time t_m when Hubble is of the order of the mass, we implicitly define $4H(t_m) \equiv m$. The overall effect of the kick is given by the ratios of these time scales and the overall strength of the forcing term.

- (A) If the source effect ends before the first oscillation $t_F \ll t_m$ the field will get suddenly displaced from its initial value, undergo some Hubble friction and finally starts to oscillate at t_m (a quick kick solution).
- (B) On the other hand, if the process of Q -number generation is slow or late $t_F \gg t_m$, the effect of the forcing term is averaged out and vanishes as can be seen in the source term in Eq. (10) where the faster time dependence of $\hat{J} \sim \cos(mt)$ makes the t'' integral cancel.

As a result, for the rest of this work we will focus on the first regime. You can see some examples of the “kick-alignment” solutions for different choices of t_Q/t_m in Fig. 1. These solutions were obtained by direct integration of the EoM. The following is a discussion of the field evolution, the reader interested in the outcome at late times can skip to Eq. (27) for the result.

While the exact solution can be given in the form of Eq. (10), the integral with \hat{J}^{-2} presents (spurious) poles when the homogeneous solution starts oscillating, which make its numerical evaluation challenging. Here instead, in order to obtain the leading parametric dependence, we will

exploit the transient nature of the forcing term and split the evolution of ϕ into three stages:

- (i) $t < t_Q$ The source term has not been “turned on” yet so the exact solution in Eq. (10) reduces to the homogeneous term $\hat{J}(t)$ as given in Eq. (11) satisfying the initial conditions at t_i .
- (ii) $t_Q < t < t_F$ While the source term is active the field evolution is given by the initial conditions and solving the double integral in Eq. (10), with $t_i = t_Q$.
- (iii) $t > t_F$ After the forcing term vanishes the evolution of $\phi(t)$ is again given by the homogeneous version of the EoM. The solution looks like Eq. (11) with different C_1 and C_2 as compared to the solution in (i). The coefficients can be obtained by matching with the results of the previous stage at $t = t_F$. The amplitude of this solution determines the relic density of ϕ .

In general, we can think about the above calculation as a scattering problem: a free wave function enters a region with nontrivial potential and the final solution is again a linear combination of free wave functions with a different amplitude and a nonzero phase-shift.

As we mentioned, given our initial conditions before Q -number generation the field can be described as:

$$(i) \quad \phi(t < t_Q) = C_1 \frac{J_{1/4}(mt)}{(mt)^{1/4}} \equiv \hat{J}_i(t), \quad (12)$$

where given our assumption of $t_F \ll t_m$ and the fact that by definition $t_Q < t_F$ one has that the homogeneous solution in this regime is a constant. In other terms, the field is Hubble stuck at early times as is familiar from the misalignment mechanism. Therefore

$$(i) \quad \phi(t < t_Q) = \phi(t_i). \quad (13)$$

During the regime (ii) the exact solution should be used and the field reads

$$(ii) \quad (t_Q < t < t_F) \quad \phi(t) = \hat{J}_i(t) \left(1 + \int_{t_Q}^t \frac{dt'}{a^3 \hat{J}_i^2(t')} \int_{t_Q}^{t'} dt'' \frac{\hat{J}_i(t'') d(a^3 n_Q)}{f dt''} \right). \quad (14)$$

This expression simplifies given that our assumption $t_F \ll t_m$ means \hat{J}_i is still to a good approximation constant given by initial conditions as in Eq. (12), and so

$$(ii) \quad \phi(t) = \phi(t_i) + \frac{1}{f} \int_{t_Q}^t n_Q(t) dt + \mathcal{O}\left(\frac{t_F}{t_m}\right) \quad (15)$$

$$\dot{\phi}(t) = \frac{n_Q(t)}{f} + \mathcal{O}\left(\frac{t_F}{t_m}\right). \quad (16)$$

This means in particular that the source has generated velocity in ϕ and there is a contribution to $Y_{1/4}$ in the matching, which to first order in each coefficient reads:

$$(iii) \quad \phi(t) = \bar{C}_1 \frac{J_{1/4}(mt)}{(mt)^{1/4}} + \bar{C}_2 \frac{Y_{1/4}(mt)}{(mt)^{1/4}} \quad (17)$$

$$\bar{C}_1 = 2^{1/4} \Gamma(5/4) (\phi(t_F) + 2t_F \dot{\phi}(t_F)) \quad (18)$$

$$\bar{C}_2 = \frac{2^{5/4} \pi}{\Gamma(1/4)} t_F \dot{\phi}(t_F) \left(\frac{t_F}{t_m} \right)^{1/2}, \quad (19)$$

where $\phi(t_F)$, $\dot{\phi}(t_F)$ are expressions in Eqs. (15) and (16) evaluated at t_F , and this function describes the remainder of the evolution of the field; in particular, at late times after horizon crossing one has

$$\phi(t \gg t_m) = \sqrt{\frac{\bar{C}_1^2 + \bar{C}_2^2}{2\pi}} \left(\frac{a(t_m)}{a(t)} \right)^{3/2} \cos(mt + \omega). \quad (20)$$

Hence our approximation $t_F/t_m \ll 1$ means that at late times (but not intermediate ones!) the amplitude is given by \bar{C}_1 .

An important fact to stress is that, provided $t_F \ll t_m$ the value of t_F at which we do the matching should not matter. This is evidenced by the fact that the dependence on t_F on \bar{C}_1 can be rewritten as

$$\begin{aligned} \phi(t_F) + 2t_F \dot{\phi}(t_F) &= \phi(t_i) + \int_{t_Q}^{t_F} \frac{n_Q(t)}{f} dt + \int_{t_F}^{\infty} \frac{n_Q(t)}{f} dt \\ &= \phi(t_i) + \int_{t_Q}^{\infty} \frac{n_Q(t)}{f} dt, \end{aligned} \quad (21)$$

where we used the fact that by t_F the time dependence of n_Q is $\propto a^{-3}$. In view of the above and substitution back in Eq. (20) via Eq. (17) one can see that the effect at late times can be encoded in a shift on the initial value of the field by an amount $\delta\phi$ given by the integral of Q abundance. While this much one could have guessed from the exact solution Eq. (10), assuming in the inner integral $a^3 n_Q$ has a step-function form and the outer integral converges due to the a^{-3} factor, this derivation as presented makes it explicit and more general.

The net effect of a shift in the field is the reason we call this mechanism the “kick-alignment”, and the fact that this shift is calculable and related to observed physics makes it more predictive than its well studied counterpart, the misalignment. Nonetheless, for the shift to dominate one has to assume that the initial value $\phi(t_i)$ is much smaller, $\phi(t_i) \ll \delta\phi$ which in certain cases might imply fine-tuning [unless a dynamic mechanism is put in play to set $\phi(t_i) = 0$ but we will not theorize about this here]. In the following, we assume $\phi(t_i) \ll \delta\phi$ holds since in the opposite case, the

dominant initial value leads to conventional and exhaustively studied misalignment.

In order to make the potential fine-tuning explicit we cast the integral for $\delta\phi$ in terms of observed quantities for clearer display of its order of magnitude

$$\delta\phi \equiv \frac{1}{f} \int n_Q(t) dt + \mathcal{O}\left(\frac{t_F}{t_m}\right) \quad (22)$$

$$\begin{aligned} &= \sqrt{\frac{\pi}{45}} \frac{Y_Q \sqrt{g_*(T_Q)} M_{\text{pl}} T_Q}{f} \int \frac{dT}{T_Q} \sqrt{\frac{g_*(T)}{g_*(T_Q)}} \frac{Y_Q(T)}{Y_Q(T_0)} \\ &+ \mathcal{O}\left(\frac{t_F}{t_m}\right) \end{aligned} \quad (23)$$

$$\equiv \sqrt{\frac{\pi}{45}} \frac{Y_Q \sqrt{g_*(T_Q)} M_{\text{pl}} T_Q}{f} I(r_m, r_\tau), \quad (24)$$

where we define $Y_Q \equiv n_Q/s$ and the last line defines $I(r_m, r_\tau)$ which includes subleading corrections and is a function of variables

$$r_m^2 = \frac{t_Q}{t_m} \quad r_\tau^2 = \frac{t_Q}{\tau}, \quad (25)$$

where we introduced τ as the typical scale of Q production, (e.g., the lifetime of a heavy particle decaying asymmetrically) and one has that $t_F - t_Q = \text{*several* } \tau$ so that the process as ended by t_F (e.g., the effect has decreased by *several* e-folds for the heavy particle decay case). The justification for this extra notation is that as underlined late time results should not depend t_F but they will depend on τ . In addition the first order correction on r_m can be captured selecting the limits of integration as shown in Appendix A.

This integral is order 1 for $\tau \sim t_Q$ so for order of magnitude estimates one can take the shift to be, in terms of the angle variable $\theta = \phi/f$:

$$\delta\theta = \frac{\delta\phi}{f} \simeq Y_Q \sqrt{g_*} T_Q \frac{M_{\text{pl}}}{f^2}. \quad (26)$$

Given the matter–antimatter asymmetry today one therefore has an order one angle displacement for $f \sim \sqrt{T_Q/\text{GeV}} 10^5 \text{ GeV}$. At the same time, we note that if the displacement of the field is large enough $\delta\phi \sim f$ one can not expand the potential to leading order and the nonlinear EoM should be solved. This regime is reached for $f^2 \sim Y_Q M_{\text{pl}} T_Q$; if f is near T_Q this implies $f \sim 10^9 \text{ GeV}$.

Plugging in the above definitions back into the late time behavior of the field in Eq. (20) reads

$$\phi(t) = \frac{\Gamma_{5/4}}{\sqrt{\pi}} \delta\phi \left[\frac{a(t_m)}{a(t)} \right]^{3/2} \cos(mt + \omega), \quad (27)$$

with $\Gamma_{5/4} \equiv \Gamma(5/4) \simeq 0.906$. The field has evolved to have an amplitude today as

$$\phi(t_0) = A(t_0) \cos(mt_0 + \omega) \quad (28)$$

$$A(t_0)^2 = \frac{\Gamma_{5/4}^2}{\pi} \frac{43}{11g_\star} \left(\frac{64\pi^3 g_\star T_0^4}{45M_{\text{pl}}^2 m^2} \right)^{3/4} (\delta\phi)^2 \quad (29)$$

$$= \frac{\Gamma_{5/4}^2}{\pi} \left(\frac{180\pi}{g_\star(t_m)} \right)^{1/4} \frac{8s_0(\delta\phi)^2}{(mM_{\text{pl}})^{3/2}}, \quad (30)$$

so the energy density, $\rho = A^2 m^2/2$, is

$$\begin{aligned} \rho_\phi &= \frac{4\Gamma_{5/4}^2 (4\pi)^{1/4} g_\star(T_Q)}{(45)^{3/4} g_\star(t_m)^{1/4}} Y_Q \sqrt{mM_{\text{pl}}} \frac{T_Q^2}{f^2} I^2 n_Q(t_0) \\ &= 0.356 \frac{g_\star(T_Q)}{g_\star(t_m)^{1/4}} Y_Q \sqrt{mM_{\text{pl}}} \frac{T_Q^2}{f^2} I^2 n_Q(t_0). \end{aligned} \quad (31)$$

Assuming $T_Q \sim f$ and $I \sim 1$, the right abundance for ϕ to make up the dark matter can be obtained when the geometric mean mass of m and M_{pl} times the asymmetry Y_Q falls around the mass of baryons themselves. Which is to say the right relic abundance is obtained for a value 5.4 ($\Omega_{dm} \sim 5.4\Omega_b$ [16]) of the ratio:

$$\begin{aligned} \frac{\rho_\phi}{m_b n_b} &= 0.356 \frac{g_\star(T_Q)}{g_\star(t_m)^{1/4}} \frac{Y_Q \sqrt{mM_{\text{pl}}}}{m_b} \frac{T_Q^2}{f^2} \frac{n_Q(t_0)}{n_b(t_0)} I^2 \\ &= \frac{0.12 g_\star(T_Q)}{g_\star(t_m)^{1/4}} \sqrt{\frac{m}{\text{GeV}}} \frac{T_Q^2}{f^2} \frac{Y_Q n_Q(t_0)}{Y_B n_b(t_0)} I^2. \end{aligned} \quad (32)$$

The fact that the mass of dark matter falls around the electroweak scale merits some mention even if we will not speculate on why this could be the case here. What does follow unambiguously from the expression above is that there is a lower limit on the mass m if this is to be the dark matter, which we take here to be given by the theory constraint $T_Q < f$ (i.e., spontaneous symmetry breaking to occur before Q generation).

The “kick-alignment” production of ϕ is then analytically calculable within the approximations here considered yet it occurs in more general circumstances. This work however aims at showing the feasibility of the mechanism rather than exhausting its possibilities so let us proceed to an analysis with explicit realizations of B and L generation. This will give physical input on T_Q and I while providing useful nontrivial consistency conditions.

B. Q -number generation mechanisms

Here we consider the possibilities of Q violation from a heavy particle decay or alternatively through sphaleron transmission from another sector.

1. Q generation in heavy particle decay

Let some heavy particle X decay out of equilibrium and in different proportion to particles and antiparticles. The departure of equilibrium sets t_Q and we parametrize the asymmetry generation as $(N_X \sim e^{-\Gamma_X t}) Y_Q \propto (1 - e^{-\Gamma_X(t-t_Q)})$ with the identification $\tau = \Gamma_X^{-1}$ in Eq. (25). The ratio of asymmetry reads then:

$$\frac{Y_Q(t)}{Y_Q(t_0)} = \beta(1 - e^{-\Gamma_X(t-t_Q)}), \quad (33)$$

where β is an order one number and accounts for redistribution factors of Q symmetry from generation to present time typically given by sphaleron processes [e.g., for $Q=L$ and leptogenesis $\beta = 1/(1 - 12/37)$]. The integral I in terms of temperature thus reads (assuming RD, $\dot{T} = -HT$):

$$I(r_\tau, r_m) = \beta \int \frac{dT}{T_Q} (1 - e^{-r_\tau^2((T_Q/T)^2 - 1)}), \quad (34)$$

where the time and temperature for the start of Q generation is determined by the particle X falling out of equilibrium, $T_Q = M_X/x_f$ with x_f given by freeze-out. The integral I can be rewritten in terms of a Gaussian integral and the exact expression is given in Appendix B. One can estimate x_f as the temperature when the inverse decay process becomes inefficient compared to Hubble, that is

$$\Gamma_X e^{-x_f} \simeq H(x_f) \quad K \equiv \frac{\Gamma_X}{H(M_X)}. \quad (35)$$

An early departure from equilibrium $x_f \sim 1$ occurs for weak coupling $K \ll 1$ or a slightly later one $x_f \sim \log(K)$ [17] occurs for strong coupling $K \gg 1$. The strong coupling larger rate however makes Q number generation resemble a step-function whereas the slower growth for $K \ll 1$ results in a smaller value for the integral. This translates into the two asymptotic behaviors

$$\begin{aligned} K \gg 1 \quad r_\tau^2 &\simeq \frac{K \log^2(K)}{2} \quad T_Q \simeq \frac{M_X}{\log(K)} \quad \frac{I}{\beta} \simeq 1 \\ K \ll 1 \quad r_\tau^2 &\simeq \frac{K}{2} \quad T_Q \simeq M_X \quad \frac{I}{\beta} \simeq \sqrt{\pi} r_\tau, \end{aligned}$$

where we took $r_m \ll 1$. Both limits $K \gg 1$ and $K \ll 1$ will be explored in the phenomenology in Sec. III.

2. Q generation by sphaleron transfer

It could be the case instead that, e.g., a L violating decay produces an initial asymmetry while $Q = B$. It would only be when sphaleron processes transfer the asymmetry to B number that ϕ starts feeling a source. Following this example the evolution in Y_B reads [18]:

$$\frac{d(Y_B)}{dt} = -\kappa \alpha_w^4 c_1 T (Y_B - c_2 Y_{B-L}) \quad (36)$$

$$c_1 = N_f^2 \frac{3}{4} \frac{22N_f + 13}{N_f(5N_f + 3)} \quad c_2 = \frac{8N_f + 4}{22N_f + 13}, \quad (37)$$

where we assume $T > T_{EWPT}$ with the generations $N_f = 3$ and the initial Y_{B-L} is taken as an input. The time scale for this transfer of asymmetry is given by the sphaleron rate, and with the identification

$$\tau^{-1} = \kappa c_1 \alpha_{ew}^4 T_Q \quad (38)$$

one has a Y_B dependence with temperature for the solution

$$Y_B = c_2 Y_{B-L} (1 - e^{-2r_\tau^2 (T_Q/T - 1)}), \quad (39)$$

taking now T_Q as the temperature when the initial $B - L$ asymmetry is generated. Hence our integral

$$I = \int \frac{dT}{T_Q} (1 - e^{-2r_\tau^2 ((T_Q/T) - 1)}), \quad (40)$$

where the ratio r_τ^2 now relates to whether sphalerons are in equilibrium or not by T_Q ; in particular the two limits for r_τ return

$$r_\tau^2 = \frac{\Gamma_{sp}(T_Q)}{2H(T_Q)} \gg 1 \quad I = 1 \quad (41)$$

$$r_\tau^2 = \frac{\Gamma_{sp}(T_Q)}{2H(T_Q)} \ll 1 \quad I = \left(\log \left(\frac{1}{2r_\tau^2} \right) - \gamma_E \right) 2r_\tau^2, \quad (42)$$

with γ_E the Euler-Mascheroni constant. The full solution without neglecting r_m is given in terms of the incomplete Gamma function in Appendix B.

The discussion of Sec. II E suggests that if sphalerons can source the asymmetry, thermal production might overcome kick-alignment and the subsequent phenomenological analysis focuses on case (I) for simplicity.

It is clear, nonetheless, that if one has a Goldstone boson for a Q symmetry there will be Q -violating-sourced production of the field. Whether this is a sufficiently strong generation mechanism is then a quantitative question.

C. Back reaction on Y_Q

A relevant point to address is the feedback into the asymmetry Y_Q of the field ϕ . The effect of a nonvanishing time derivative of ϕ creates a nonzero chemical potential for the fermions that make up J_Q as can be seen in Dirac's equation:

$$[i\mathcal{D} - (m + \gamma^0 Q_\psi \dot{\phi}/f)]\psi = 0. \quad (43)$$

This is the principle behind spontaneous baryogenesis [10,11,19]. Nevertheless this contribution by itself is not observable [as again the field rotation $\psi \rightarrow e^{-iQ\phi/f}\psi$ from (6) exemplifies], the other necessary physics for an effect is an explicit Q -violation. Through this Q -violating effect, the potential $\mu_\phi = \dot{\phi}/f$ will contribute to a particle-antiparticle imbalance. For an estimate of the effect we compare this generated chemical potential with the potential that produced the asymmetry in the first place,

$$\frac{\mu_0}{T} \simeq \frac{n_Q}{n_q} \quad \frac{\mu_\phi}{T} \simeq \frac{\dot{\phi}}{fT} \simeq \frac{n_Q}{f^2 T}, \quad (44)$$

where we used $n_Q = n_q - n_{\bar{q}}$, and so $\mu_0 > \mu_\phi$ provided $f > T$, and hence the backreaction from the presence of ϕ is negligible by default because this is the condition we imposed already so that the theory contains a Goldstone boson ϕ by the time the Universe reaches temperature T_Q .

Nevertheless, we believe that the scenario in which the feedback from the fermions is in some sense in equilibrium with the effect we describe here could be of theoretical interest and leave this possibility for future study.

D. Contrast with other production mechanisms

Below we compare the parametric dependence of various DM production mechanisms: freeze-out (FO), freeze-in (FI), the misalignment (MA) and the kick-alignment (KA). Taking the yield Y as $\rho/(mT^3)$ one has the estimates for each mechanism as

$$\begin{aligned} \text{FO: } Y &\propto \frac{1}{\langle \sigma v \rangle M_{\text{pl}} m} & \text{FI: } Y &\propto \alpha^2 \frac{M_{\text{pl}}}{m} \\ \text{MA: } Y &\propto \frac{\theta_0^2 f^2}{M_{\text{pl}} \sqrt{M_{\text{pl}} m}} & \text{KA: } Y &\propto Y_Q^2 \frac{T_Q^2}{f^2} \sqrt{\frac{M_{\text{pl}}}{m}}, \end{aligned}$$

where we took $n_Q/T^3 \sim Y_Q$ in Eq. (31) which gives an extra power of Y_Q . The arrangement is meant to be meaningful: we have that in the first row abundance is given by interactions with the thermal bath as described in Boltzmann's equations, whereas abundance in the second line is determined by the field equation of motion. The second column has a yield proportional to the coupling, the first column's yield is inversely proportional to the coupling. In this sense it is clear that the kick-alignment completes the square.

Notice that freeze-in operates in situations where the initial population can be neglected, as opposed to the freeze-out, and hence can work with small couplings. The same way the kick-alignment works best when the initial field value is small, exactly opposite to the case of misalignment. In that sense the kick-alignment is to misalignment as freeze-in is to freeze-out.

E. Thermal production

The coupling of ϕ can be reshuffled to baryon or lepton violating interactions via a ϕ -dependent symmetry transformation, as shown in Eqs. (6) and (7). Since the SM has no relevant B or L violating operators, the Q -violating interactions have to be nonrenormalizable at energies around the Standard Model. These type of interactions induce thermal production dominated by the UV scale. This introduces model dependence yet it also means that the field is easy to produce at early times whereas at low energies its couplings are very suppressed.

There is one such production channel with $B + L$ violation in the SM in sphaleron processes so let us begin by estimating this.

The temperature dependence of the rate for sphaleron distinguishes two regimes. After electro-weak symmetry breaking the rate is dominated by an exponential of minus the sphaleron energy over the temperature whereas at higher temperatures the tunneling suppression is lifted and the rate per unit volume scales with

$$\frac{\Gamma_{\text{sp}}}{V} \sim \kappa(\alpha_w T)^4 \quad T \gg T_{\text{EWPT}} \quad (45)$$

if we rotate the fermions as in Eq. (6) and (7) to cancel the coupling $\partial_\mu \phi J^\mu$ the field will appear instead on the 't Hooft vertex as a phase factor, which, when expanded, yields a linear coupling of ϕ/f as sketched in Fig. 2. Assuming then the reheat temperature is greater than a few TeV, $f \gg T$, we expect the thermal production to be dominated by the sphaleron processes so

$$a^3 \frac{da^3 n_\phi}{dt} \simeq \kappa(\alpha_w T)^4 \left(\frac{T}{4\pi f} \right)^2; \quad (46)$$

assuming the field was initially out of equilibrium and that it never reaches it, we are in the regime of UV freeze-in (otherwise we have to include the inverse processes as well). In terms of temperature this thermal production reads ($Y_\phi = n_\phi/T^3$):

$$\frac{dY_\phi}{dT} \simeq -\sqrt{\frac{45}{4\pi^3 g_\star}} \frac{\kappa \alpha_w^4 M_{\text{pl}}}{(4\pi f)^2}, \quad (47)$$

which confirms the yield is dominated by the UV (high T). This thermal contribution to energy density is then:

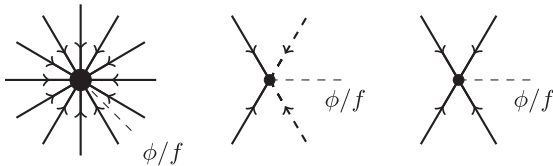


FIG. 2. ϕ coupling for thermal production via sphalerons, dimension 5 (\mathcal{L}), and 6 (\mathcal{B}) operators.

$$\rho_\phi \simeq m_\phi \frac{43}{11g_\star} \sqrt{\frac{45}{4\pi^3 g_\star}} T_0^3 \frac{\kappa \alpha_w^4 M_{\text{pl}} T_{rh}}{(4\pi f)^2}. \quad (48)$$

Comparison with kick-alignment production shows how the different scaling with m implies that for heavy m this thermal mechanism dominates. This poses a strong constraint on successful kick-alignment. This can be avoided taking our symmetry as $Q = B - L$ or any other combination to be preserved in the SM.

Other thermal channels of ϕ production will equally be dominated by high temperature but the specifics follow the mechanism by which the baryon asymmetry of the Universe was generated. To estimate this production let us consider the leading operators violating L and B are:

$$C_5 \mathcal{O}_5 + C_6 \mathcal{O}_6 = \frac{y_N^2}{2M_N} (\ell H)^2 + \frac{y_X^2}{M_X^2} qq q \ell. \quad (49)$$

These will produce the vertices for ϕ interaction as shown in Fig. 2. This simplistic picture still allows for nontrivial constraints on kick-alignment production once the couplings and masses in these operators are identified with the Q generation mechanism.

The thermal production of ϕ from these nonrenormalizable operators reads c.f. [20];

$$\frac{dY_\phi}{dT} = -\sqrt{\frac{45}{4\pi^3 g_\star}} \frac{M_{\text{pl}}}{16\pi^5 (4\pi f)^2} (C_d T^{d-4})^2. \quad (50)$$

In particular with the definition of the operators above,

$$Y_\phi \simeq \sqrt{\frac{45}{4\pi^3 g_\star}} \frac{y_N^4 M_{\text{pl}} M_N}{3 \times 64\pi^5 (4\pi f)^2} \sin^2 \alpha \quad (51)$$

$$Y_\phi \simeq \sqrt{\frac{45}{4\pi^3 g_\star}} \frac{y_X^4 M_{\text{pl}} M_X}{5 \times 16\pi^5 (4\pi f)^2} \sin^2(\alpha - \pi/4) \quad (52)$$

were $v = 174$ GeV. If one assumes \mathcal{O}_5 is also the sole contribution to light neutrino masses the relation $m_\nu = y_N^2 v^2 / M_N$ holds and can be used to further constrain the parameter space. Finally we note that other couplings such as $\phi^2 H^2$ [21] will be induced at loop level yet they will be suppressed by the Q violation parameter in the absence of which ϕ presents a shift symmetry.

III. REALIZATION

In order to evaluate the feasibility of the kick-alignment production mechanism proposed in the previous section we will explore particular cases of leptogenesis and baryogenesis. In particular we will consider the two scenarios with the symmetry Q as

$$Q = B - 3L_e.$$

This choice implies the symmetry is respected by the Standard model and hence no sphaleron thermal production, while the reason for selecting a flavored symmetry is phenomenological and laid out in each section below.

A. Grand unified theory-like baryogenesis

In this case the baryon violating operator is the standard $\mathcal{O}_6 = qqql$, where we take the strength of the operator to be $y_X^2 \sim M_X/M_{\text{pl}}$ which implies small $K = \Gamma_X/H_X \sim 1/4\pi\sqrt{g_*}$. The bound on baryon number violation processes from proton decay $y_X^2/M_X^2 \leq 1/(10^{16} \text{ GeV})^2$ can be combined to give $M_X \geq 10^{13} \text{ GeV}$. This very stringent bound from proton decay implies both that the lifetime of ϕ is well beyond the age of the universe and the thermal production through the operator \mathcal{O}_6 is suppressed up to high masses m .

As a result, the allowed range of parameters for which ϕ is produced through kick-alignment is large and easily exceeds the thermal generation. Most grand unified theories (GUT) nonetheless respect $B - L$ (every two-SM-fermion combination has the same $B - L$ number, modulo hermitian conjugate terms) even if they break B which means that no kick would be produced in the decay of the heavy states for $Q = B - L$ (selecting B turns on sphaleron thermal production which typically overpowers KA); which is the reason we select a lepton-flavored which is generically broken in the decay yet respected in the SM, $Q = B - 3L_e$. The main constraints are then: a) upper bound on f from nonperturbative gravitational contributions to the mass m , b) the combination of $y_X^2 \sim M_X/M_{\text{pl}}$, c) proton lifetime bound for a lower bound on f , and d) the overtaking of thermal contributions for large m .

For f close to the Planck scale the gravitational non-perturbative contributions to the mass of ϕ as taken from [22] are sizable and imply a $f - m$ correlation as shown by the yellow line on Fig. 3. This is to be taken as a conservative upper limit in the absence of a radial mode; its inclusion generically makes the bound stronger. Alternatively, a point lying on this line can be taken as a dark matter candidate whose mass is purely gravitational. It is however important to emphasize that this is a “theory bound” as opposed to an experimental bound.

The lower bound on f follows from fitting requiring correct relic abundance (a sum of kick-alignment and thermal production) together with proton lifetime constraints. Where the kick-alignment dominates over thermal production the relation $T_Q \propto M_X$ implies that for a given mass m , f cannot be lower than $\sim T_Q \sqrt{m/\text{GeV}} \simeq M_X \sqrt{m/\text{GeV}}$ or the abundance of ϕ would be larger than observed. This bound is shown by the blue-shaded region in Fig. 3. In the red region thermal production overcomes kick-alignment production. Finally, the low mass limit

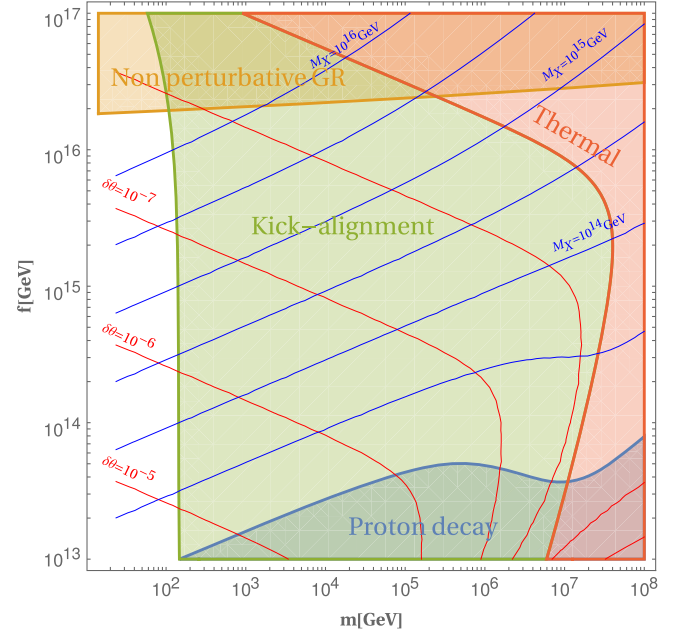


FIG. 3. GUT-like asymmetry generation with $y_X^2 \sim M_X/M_{\text{pl}}$ and $Q = B - 3L_e$. See text and labels for details.

comes from requiring a good EFT expansion $f > T_Q$. Lines of constant field shift and heavy particle mass M_X are displayed in red and blue, respectively, and extend to the left as far as $\pi f/2 > T_Q$. Note, that this internal consistency condition is, as parametrized in our plot, somewhat sensitive to the exact choice of the boundary because $\rho_\phi = \text{const} \propto m^{1/2} f^{-2}$, and so requiring $\pi f > T_Q$ instead changes the “allowed” mass range by a factor of 16. However, this is to be expected, because at this point additional degrees of freedom are relevant and the calculation is unreliable without a full model.

B. Leptogenesis

For leptogenesis the field ϕ resembles the Majoron as DM [23–25], albeit with a different production mechanism given that typically conventional freeze-in is considered [26].

We assume that the heavy neutrinos that give rise to the asymmetry also produce the active neutrino masses for a more constrained scenario. The coupling of the field ϕ through the lepton-number violation operator \mathcal{O}_5 is then given by neutrino masses and the effective scale is now much lower than for Baryon number violation through \mathcal{O}_6 . This implies stronger constraints from thermal production and from the cosmic scale stability of ϕ . One finds conversely that now $K \sim y_N^2 M_{\text{pl}}/M_N \sim M_{\text{pl}} m_\nu/v^2 \gg 1$ which yields an $I \approx 1$.

The lifetime and thermal production of ϕ are related to neutrino masses by the effective neutrino mass m_ν^{eff} that the lepton part of our symmetry aligns with. This is $(m_\nu^{\text{eff}})^2 = \sum m_{\nu_i}^2$ for flavor blind lepton number. This possibility seems in tension with the constraints so instead

we select the symmetry $3L_e$ which, via a mild suppression in m_ν^{eff} , allows successful kick-alignment production. However, there is not much room for evading constraints, since in the decoupling limit the kick-alignment production vanishes. This in turn, leads to testable consequences and provides a well defined ballpark for f and m . For the present symmetry in the lepton sector $B - 3L_e$, one has

$$(m_\nu^{\text{eff}})^2 = \frac{1}{2} |3U_{ei}^2 m_i|^2 + \frac{1}{2} \sum_\beta |3U_{ei} U_{\beta i} m_{\nu_i}|^2. \quad (53)$$

We adopt normal hierarchy and a massless first generation neutrino. This implies the contribution from the heaviest neutrino $m_{\nu_3} \sim \Delta m_{\text{atm}}$ suppressed by θ_{13} . This suppression can be further aided by a selection of the Majorana phase α in

$$m_\nu = \text{Diag}(0, m_{\nu_2} e^{i\alpha}, m_{\nu_3}) \quad (54)$$

of $\alpha = -0.6$. However, this does not lead to significant fine-tuning. All in all, this means $m_\nu^{\text{eff}} \simeq 0.017$ eV taking central values for neutrino mass parameters from [27]. For reference this gives a prediction for neutrino-less beta decay $m_{\beta\beta} = 3$ meV.

The lifetime relative to the age of the universe in terms of this effective mass

$$\frac{\tau_U}{\tau_\phi} = 0.33 \frac{(m_\nu^{\text{eff}})^2}{\Delta m_{\text{atm}}^2} \left(\frac{10^{10} \text{ GeV}}{f} \right)^2 \frac{m}{\text{GeV}}. \quad (55)$$

This ratio has to be smaller than one by at least some five orders of magnitude [28] which imposes a lower bound on f shown as the blue-shaded region in Fig. 4. In addition for kick-alignment production to dominate over thermal production there is an upper bound on f , similarly to the GUT baryogenesis scenario. This follows, from the kick-alignment relic abundance scaling with heavy mass as $(M_N/f)^2 \sqrt{m/\text{GeV}}$ whereas the thermal contribution substituting $y_N^2 \sim M_N m_\nu^{\text{eff}}$ scales then with M_N^3 . For fixed m larger f , the model requires larger M_N and thermal production overtakes kick-alignment in the red-shaded area of Fig. 4. In addition the already mentioned consistency condition $f > T_Q$ sets a lower bound on the mass m if ϕ is to be the dark matter. Lines for constant shift and mass M_N are shown in red and blue and extend to low masses up to $\pi f/2 > T_Q$. These constraints are shown to narrow down the allowed parameter space in Fig. 4 to a region around $m \sim 100$ MeV, $f \sim 3 \times 10^{11}$ GeV.

Finally, some notes on the detection of such candidate. A distinctive signal would be present in B or L violating interactions and decays. The strong bounds on lifetime nevertheless make these events rare and hard to capture in, say, Hyper-Kamiokande where we can estimate that in the next 30 years the probability to see one such decay to be at the level of 10^{-3} .

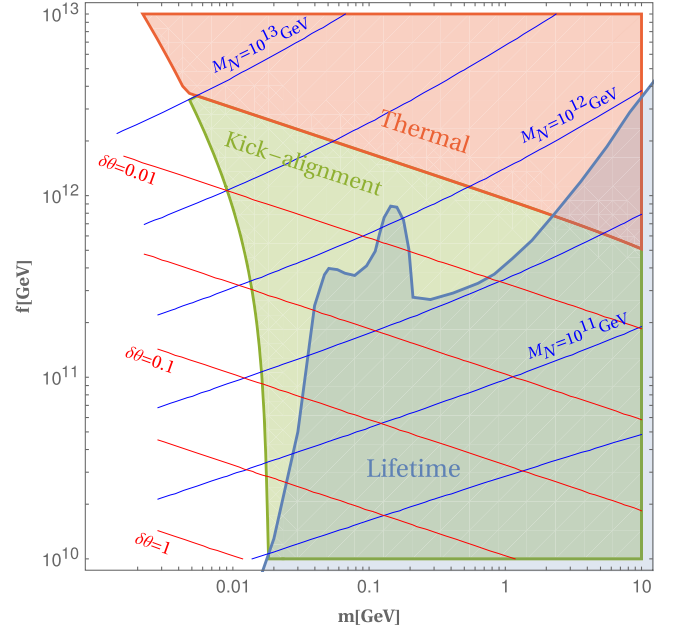


FIG. 4. Leptogenesis scenario with large $K \sim M_{\text{pl}} m_\nu / v^2$ and $Q = B - 3L_e$; see text for details.

The rarity of such processes can be understood because the bounds on lifetime of ϕ come from processes in which ϕ decays into neutrinos, which are subsequently detected on Earth. On the other hand, in order to obtain the smoking gun signal, we need to observe an event in which ϕ decays in the detector itself—a measurement that suffers from ratio of volume of the detector to the volume of the local dark matter density.

On the other hand, the resemblance of a Majoron means that phenomenological searches for this particle would apply to our candidate as well; see e.g., [29]. We leave the exploration of other possibilities for its detection like studying the effects on matter of a ϕ background to future study.

IV. CONCLUSIONS

This work laid out the possibility of primordial plasma kicking the Goldstone boson of a SM-preserved global symmetry as a mechanism to produce dark matter density.

We have shown that the generation of the matter–anti-matter asymmetry acts as a source in the evolution of the Goldstone field. This contribution produces a displacement in the field at early times which sets the amplitude for oscillations when the zero mode enters the horizon and behaves like cold matter. The mechanism can then be described as dynamically setting the initial conditions for misalignment dark-matter production which nonetheless yields a qualitatively different prediction for relic abundance similar to how freeze-in differs from freeze-out.

This production mechanism was studied in conjunction with different baryo/leptogenesis models and we find it is a

feasible possibility for dark matter production. In particular, combination with leptogenesis seems to offer viable parameter space for dark matter production with mass 10 MeV–1 GeV and decay constant f in the range of 10^{10} – 10^{12} GeV.

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APPENDIX A: MATCHING AFTER THE KICK

After Q -number generation the source term has been turned off and a homogeneous solution describes the evolution. One can then take $\phi(t_F > t_Q + \tau)$

$$\phi(t_F) = \bar{C}_1 \frac{J_{1/4}(mt_F)}{(mt_F)^{1/4}} + \bar{C}_2 \frac{Y_{1/4}(mt_F)}{(mt_F)^{1/4}} \quad (\text{A1})$$

and match with initial condition equation (10) and its derivative evaluated at t_F , assuming $mt_F \ll 1$, this returns

$$\bar{C}_1 = 2^{1/4} \Gamma_{5/4} \left(\phi(t_F) + 2t_F \dot{\phi}(t_F) \left(1 + \frac{\Gamma_{-1/4} \sqrt{mt_F}}{2\Gamma_{1/4}} \right) \right) + \mathcal{O}(mt_F) \quad (\text{A2})$$

$$\bar{C}_2 = \frac{2\pi(mt_F)^{1/2}}{2^{1/4}\Gamma(1/4)} t_F \dot{\phi}(t_F) + \mathcal{O}(mt_j), \quad (\text{A3})$$

where ϕ and $t_F \dot{\phi}$ are comparable. The late time behavior reads instead $t \gg m^{-1}$:

$$\phi(t) \simeq \sqrt{\bar{C}_1^2 + \bar{C}_2^2} \sqrt{\frac{2}{\pi(mt)^3}} \cos(mt + \omega) \quad (\text{A4})$$

so given the hierarchy $\bar{C}_1 \gg \bar{C}_2$, and in terms of t_m one obtains relation in Eq. (27). In particular the 0th and first correction $\mathcal{O}(\sqrt{mt_F})$ come from \bar{C}_1 (from the cross term in the square as opposed to C_2^2 which starts at order mt_j) which can be written to first order in a t_F independent form as

$$\begin{aligned} \int_{t_Q}^{t_m^*} \frac{n_Q}{f} dt' &= \int_{t_Q}^{t_F} \frac{n_Q(t)}{f} dt' + \frac{n_Q(t_F)}{f} \int_{t_F}^{t_m^*} \left(\frac{t_F}{t} \right)^{3/2} dt' \\ &= \phi(t_F) + 2t_F \dot{\phi}(t_F) \left(1 - \frac{\sqrt{t_F}}{\sqrt{t_m^*}} \right) \end{aligned} \quad (\text{A5})$$

so direct comparison with Eq. (A2) and the fact that $\Gamma_{-1/4}/\Gamma_{1/4} \simeq -1.35 < 0$ allows to write \bar{C}_1 in a t_j independent form

$$\bar{C}_1 = 2^{1/4} \Gamma_{5/4} \int_{t_Q}^{t_m^*} \frac{n_{\Delta Q}(t)}{f} dt + \mathcal{O}(mt_j) \quad (\text{A6})$$

$$t_m^* = \left(\frac{\Gamma_{1/4}}{\Gamma_{-1/4}} \right)^2 \frac{4}{m} \simeq \frac{2.188}{m} \simeq 1.1 r_m^{-2} t_Q, \quad (\text{A7})$$

which is the field displacement as in Eq. (22) for late enough t or rewritten in terms of temperature in Eq. (23).

APPENDIX B: INTEGRALS

The integral of Q -number abundance can be cast into an integral over temperature and using the dimensionless variable $r = T/T_Q$ one can factor out the typical scale of the process to leave a dimensionless function of r_τ , r_m . The dependence on r_m , to first order, can be captured in the limits of the integral with cf. (A7)

$$r_m^* = \sqrt{\frac{t_Q}{t_m^*}} = \frac{|\Gamma(-1/4)|}{\Gamma(1/4)} \frac{\sqrt{t_Q m}}{2} \simeq 0.957 r_m. \quad (\text{B1})$$

The integral of Eq. (34) reads

$$\begin{aligned} I &= \int_{r_m^*}^1 dr (1 - e^{-r_\tau^2(r^{-2}-1)}) \\ &= r_\tau e^{r_\tau^2} \left(\Gamma\left(\frac{1}{2}, r_\tau^2\right) - \Gamma\left(\frac{1}{2}, \frac{r_\tau^2}{(r_m^*)^2}\right) \right) \end{aligned} \quad (\text{B2})$$

$$-r_m^* (1 - e^{r_\tau^2(1-(r_m^*)^{-2})}), \quad (\text{B3})$$

while that of Eq. (40) reads

$$\begin{aligned} I &= \int_{r_m^*}^1 dr (1 - e^{-2r_\tau^2(r^{-1}-1)}) \\ &= 2r_\tau^2 e^{2r_\tau^2} \left(\Gamma(0, 2r_\tau^2) - \Gamma\left(0, \frac{2r_\tau^2}{r_m^*}\right) \right) \end{aligned} \quad (\text{B4})$$

$$-r_m^* (1 - e^{2r_\tau^2(1-(r_m^*)^{-1})}), \quad (\text{B5})$$

where $\Gamma(x, y)$ is the incomplete Gamma function:

$$\begin{aligned} \Gamma(x, y) &= \int_y^\infty z^{x-1} e^{-z} dz \\ \Gamma(x, y \gg 1) &\simeq y^{x-1} e^{-y} \quad \Gamma(x, y \ll 1) \simeq \Gamma(x) - \frac{y^x}{x}. \end{aligned} \quad (\text{B6})$$

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